Alpha-decay lifetimes semiempirical relationship including shell effects

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Abstract

A new version of the semiempirical formula based on fission approach of alpha decay is derived, by using the optimum values of the fitting parameters determined for even-even nuclei, combined with hindrance factors for even-odd, odd-even, and odd-odd nuclides. The deviations from experimental data for two regions of nuclear chart (493 alpha emitters with $Z = 52 - 118$ and 142 transuranium nuclei including superheavies ($Z = 92 - 118$), respectively) are compared with those obtained by using the universal curve and the Viola-Seaborg semiempirical relationship.

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The existence of a fission barrier produced by shell effects in the region of superheavy nuclei [1, 2] continues to be proved by the successful synthesis of the heaviest nuclides [3, 4, 5], which stimulate a corresponding theoretical development (see [6, 7, 8, 9] and the references therein). In 1969 it was shown that fission barriers as high as 8 to 12 MeV develop due to the shell corrections around double shell closures which appeared in the standard shell model at $Z = 114$, $N = 184$. Nowadays some other magic numbers have been proposed, e.g. $Z = 126$. The majority of these proton-rich nuclei are mainly decaying by α-particle emission, motivating an important effort to derive different methods allowing to estimate the half-lives [10, 11] and to perform systematic studies in this region e.g. [12, 13, 14].

Three such methods based on fission theory extended to a very large asymmetry (the analytical superasymmetric fission (ASAF) model [15, 16], the universal (UNIV) curve [17], and the semiempirical relationship (SemFIS) [18, 19]) have been compared recently [20, 21]. The kinetic energy of the emitted particle, $E_\alpha = QA_1/A$, is given at the input. It was shown that the description of data in the vicinity of the magic proton and neutron numbers, where the errors of the other relationships are large, was improved by introducing the SemFIS formula, in which the action integral is multiplied by a second order polynomial, $\chi = \chi(x, y)$, in the variables x and y expressing the "distance" from the nearest magic plus one neighbours $x = (N - N_i)/(N_{i+1} - N_i)$ and $y = (Z - Z_i)/(Z_{i+1} - Z_i)$, where $N_i > N > N_{i+1}$ and $Z_i > Z > Z_{i+1}$. In this way not only the Z dependence (present in the majority of semiempirical relationships) but also the strong influence of the neutron shell effects are taken into account. A computer program [19] allows to change automatically the fit parameters, every time a better set of experimental data is available. This polynomial approximation needs six parameters in each group of even-even, even-odd, odd-even and odd-odd alpha emitters. The purpose of the present work is to derive an alternative with less number of parameters and to compare the results with those obtained by using the universal curve. The same six quantities determined for even-even nuclei should be used in the other groups, only completed in the expression of $\log T$ by a different hindrance factor.

We study a spontaneous process of α-decay in which a *parent* nucleus A, Z in its ground state is split into two fragments $A_1 = A - 4, Z_1 = Z - 2$ and $A_2 = 4, Z_2 = 2$

$$AZ \rightarrow \ {}^{A_1}Z_1 + \ {}^{A_2}Z_2 \tag{1}$$

in a way that conserves the hadron numbers $A = A_1 + A_2$, $Z = Z_1 + Z_2$. The heavy fragment

A_1, Z_1 is called *daughter* and the light one *emitted α-particle*. The partial decay half-life T of the parent nucleus is related to the disintegration constant λ of the exponential decay law in time by the relationship

$$\frac{\ln 2}{T} = \lambda \tag{2}$$

The quantum mechanical tunnelling process leads to a relatively simple expression for λ as the product of the three model dependent quantities

$$\lambda = \nu S P \tag{3}$$

where ν is the frequency of assaults, S is the preformation probability of the α-particle at the nuclear surface, and P is the penetrability of the external part of the barrier [15, 17].

Very frequently the penetrability is calculated by using the one-dimensional Wentzel–Kramers–Brillouin (WKB) approximation

$$P = \exp(-K); \quad K = \frac{2}{\hbar}\int_{R_a}^{R_b} \sqrt{2B(R)E(R)}dR \tag{4}$$

where B is the nuclear inertia, approximated by the reduced mass $\mu = mA_1A_2/A$, m is the nucleon mass, E is the potential energy from which the Q-value has been subtracted out, R_a and R_b are the classical turning points, and K is the action integral.

The equations 2-4 are used as a starting point by the three methods mentioned above (ASAF, UNIV, and SemFIS) based on fission theory of α-decay. The zero point vibration energy is $E_v = h\nu/2$ in which h is the Plank constant. The outer potential barrier is of Coulomb nature, $E(R) = e^2 Z_1 Z_2/R - Q$, with the electron charge $e = 1.43998$ MeV·fm. The other numerical constants are given by $(1/2)h \ln 2 = 1.4333 \cdot 10^{-21}$ MeV·s and $2\sqrt{2m}/\hbar = 0.43921$ MeV$^{-1/2}$fm$^{-1/2}$.

It is instructive to outline the derivation of the UNIV formula. For α-decay of even-even nuclei the eqs. 2-4 allow us to obtain a simple relationship of the decimal logarithm of the half-life

$$\log T = -\log P + c_{ee} \tag{5}$$

where the additive constant $c_{ee} = \log S_\alpha - \log \nu + \log(\ln 2) = -20.325$ if we are making two approximations: $S_\alpha = 0.0180302$ and $\nu = 10^{22.01}$ s^{-1}. For even-odd, odd-even and odd-odd nuclei we replace c_{ee} by $c_{eo} = c_{ee} + h_{eo}$, $c_{oe} = c_{ee} + h_{oe}$, and $c_{oo} = c_{ee} + h_{oo}$, respectively, where h_{eo}, h_{oe}, h_{oo} are the mean values of the hindrance factors in these groups of nuclides.

In a doubly logarithmic scale the equation 5 represents a straight line with a slope equal to unity. The penetrability of an external Coulomb barrier, having as the first turning point the separation distance at the touching configuration $R_a = R_t = R_1 + R_2$ and the second one defined by $e^2 Z_1 Z_2 / R_t = Q$, may be found analytically as

$$-\log P = 0.22873(\mu_A Z_1 Z_2 R_b)^{1/2} \left[\arccos\sqrt{r} - \sqrt{r(1-r)} \right] \tag{6}$$

where $r = R_t/R_b$, $R_t = 1.2249(A_1^{1/3} + A_2^{1/3})$, and $R_b = 1.43998 Z_1 Z_2/Q$. We use the liquid drop model radius constant $r_0 = 1.2249$ fm.

A great number of alpha emitters are known [12, 13, 22, 23, 24, 25] as it may be seen in figure 1 where we plotted the experimental half-lives of 162 even-even, 122 even-odd, 115 odd-even, and of 94 odd-odd nuclides versus the neutron number of the daughter nucleus. The very strong neutron shell effect is clearly seen at the magic daughter number $N_d = 126$, where the shortest half-lives have been measured.

The corresponding universal curves for α-decay of the same nuclides as in figure 1 in four groups of even-even, even-odd, odd-even and odd-odd nuclei are displayed in figure 2. The following hindrance factors have been used: $h_{oe} = 0.445$, $h_{e0} = 0.294$, $h_{oo} = 0.842$, for even-odd, odd-even, and odd-odd nuclei, respectively. Qualitatively one can see that the data for even-even nuclei are well described by the universal curve. We can evaluate the standard root-mean-square (rms) deviation of $\log T$ values:

$$\sigma = \left\{ \sum_{i=1}^{n} [\log(T_i/T_{exp})]^2/(n-1) \right\}^{1/2} \tag{7}$$

leading to $\sigma = 0.352, 0.612, 0.546, 0.841$ for the 162 even-even, 122 even-odd, 115 odd-even, and of 94 odd-odd nuclides, respectively.

Despite the fact that a main part of the strong shell effect present in the Q-value was accounted for, there are still some systematic errors in the vicinity of the magic number of neutrons as may be seen in the plot of $\log T - \log T_{exp}$ against the neutron number of the daughter (see figure 14 of the ref. [20]).

The same kind of underestimation around neutron magic numbers is also present in the semiempirical formulae. A typical example is the Viola-Seaborg [26] semiempirical relationship

$$\log T = (aZ - b)Q^{-1/2} - (cZ + d) \tag{8}$$

We shall use for $Z \leq 82, N \leq 126$ the parameter values $a = 2.42151$; $b = 62.3848$; $c = 0.59015$, $d = 4.2109$, and for $Z > 82, N > 126$ the recently published [11] ones $a = 1.3892$; $b = -13.862$; $c = 0.1086$, $d = 41.458$. It gives excellent agreement in the region of actinides but it underestimates the lifetimes of lighter nuclei. It is very frequently used, particularly in the region of superheavy nuclei.

The behaviour around magic numbers can be improved by using the SemFIS formula in which we explicitly take into account the shell effect by multiplying the action integral, K, with the polynomial $\chi = \chi(x, y)$ mentioned above:

$$\log T = 0.43429\chi(x, y) \cdot K - 20.446 + H^f \tag{9}$$

where H^f is a hindrance factor which takes different values $H^f_{ee} = -0.025$ for even-even emitters, $H^f_{eo} = 0.420$ for e-o, $H^f_{oe} = 0.280$ for o-e, and $H^f_{oo} = 0.810$ for o-o ones.

$$\begin{aligned} K &= 2.52956 Z_1 [A_1/(AQ)]^{1/2} [\arccos \sqrt{r} - \sqrt{r(1-r)}] \; ; \\ r &= 0.423 Q (1.5874 + A_1^{1/3})/Z_1 \end{aligned} \tag{10}$$

The numerical coefficient χ, close to unity, is a second-order polynomial

$$\chi = B_1 + x(B_2 + xB_4) + y(B_3 + yB_6) + xyB_5 \tag{11}$$

with coefficients obtained by fitting the data plotted in figure 1 for 163 even-even emitters: $B_1 = 0.987389$, $B_2 = 0.009520$, $B_4 = 0.033711$, $B_3 = 0.035249$, $B_6 = -0.038941$, $B_5 = 0.019451$. The reduced variables are defined

$$x \equiv (N - N_i)/(N_{i+1} - N_i) \; ; \; N_i < N \leq N_{i+1} \tag{12}$$

$$y \equiv (Z - Z_i)/(Z_{i+1} - Z_i) \; ; \; Z_i < Z \leq Z_{i+1} \tag{13}$$

with $N_i = ..., 51, 83, 127, 185, 229, ...$, $Z_i = ..., 29, 51, 83, 115, ...$ hence for the region of superheavy nuclei $x = (Z - 83)/(127 - 83)$, $\; y = (N - 127)/(185 - 127)$.

The improvement can be seen by comparing the standard deviations of UNIV and SemFIS for the large set of data in table I. The figure 3 shows that there are nolonger large systematical discrepancies around magic numbers, because shell effects were taken into account by SemFIS. The example are given for the best results obtained in the region of even-even (top) and odd-even (bottom) nuclei.

TABLE I: Standard deviations for semiempirical formula, and universal curves in the region of all alpha emitters ($Z = 52 - 118$).

n	parity	σ_{univ}	σ_{semFIS}
162	e-e	0.352	0.233
122	e-o	0.612	0.515
115	o-e	0.546	0.497
94	o-o	0.841	0.709

In order to illustrate the possibility of using a different set of parameter values, we apply the semFIS formula for transuranium nuclei (the set of 47 even-even ($Z = 92, 118$), 45 even-odd ($Z = 92, 114$), 25 odd-even ($Z = 93, 115$), 25 odd-odd ($Z = 93, 113$) nuclides). Now we obtain the following parameters: $B_1 = 0.985415$, $B_2 = 0.102199$, $B_4 = -0.832081$, $B_3 = -0.024863$, $B_6 = -0.681221$, $B_5 = 1.50572$ and $H^f_{ee} = 0$, $H^f_{eo} = 0.63$, $H^f_{oe} = 0.51$, $H^f_{oo} = 1.26$.

The optimum hindrance factors for universal curves are $h_{ee} = 0.100$, $h_{eo} = 0.792$, $h_{oe} = 0.642$, and $h_{oo} = 1.512$.

For Viola-Seaborg besides $a = 1.3892$, $b = -13.862$, $c = 0.1086$, $d = 41.458$ we introduce the additive hindrance constants CV of -0.073 for e-e, 0.484 for e-o, 0.478 for o-e, and 1.078 for o-o nuclei.

TABLE II: Standard deviations for semiempirical formula, and universal curves in the region of transuranium alpha emitters including superheavies ($Z = 92 - 118$).

n	parity	σ_{VS}	σ_{univ}	σ_{semFIS}
47	e-e	0.282	0.267	0.164
45	e-o	0.562	0.554	0.540
25	o-e	0.608	0.543	0.514
25	o-o	0.597	0.456	0.492

From the table II one can see how the universal curves reproduce better than the Viola-Seaborg formula the experimental data. A similar property is exhibited by the semFIS formula, which behaves very well for even-even alpha emitters. Moreover, the dependence on the proton and neutron magic numbers of the semiempirical formula may be exploited to obtain informations about the values of the magic numbers which are not well known until now.

In conclusion the new version of the SemFIS formula, having a smaller number of fitting parameters, gives slightly better results than the universal curve, both for a large number (493) of alpha emitters in the whole nuclear chart ($Z = 52 - 118$) and for 142 transuranium nuclei including superheavies ($Z = 92 - 118$).

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[1] W. Greiner, *Internat. J. Modern Phys.* **5**, 1 (1996).

[2] U. Mosel and W. Greiner, *Z. Phys.* **222**, 261 (1969)

[3] S. Hofmann and G. Münzenberg, *Rev. Mod. Phys.* **72**, 733 (2000).

[4] Y. T. Oganessian et al. *Phys. Rev.* **C 72**, 034611 (2005).

[5] K. Morita et al. *J. Phys. Soc. Japan* **73**, 2593 (2004).

[6] P. Mohr, *Phys. Rev.* **C 73**, 031301 (2006).

[7] Chang Xu and Zhongzhou Ren, *Phys. Rev.* **C 73**, 041301 (2006).

[8] R. A. Gherghescu and Y. Nagame, *Phys. Rev.* **C 74**, 014611 (2006).

[9] R. A. Gherghescu and N. Carjan *Phys. Rev.* **C 71**, 054612 (2005).

[10] G. Royer, *J. Phys. G: Nucl. Part. Phys.* **26**, 1149 (2000).

[11] A. Parkhomenko and A. Sobiczewski, *Acta Phys. Pol.* **B 36**, 3095 (2005).

[12] M. Gupta and T. W. Burrows, *Nuclear Data Sheets* **106**, 251 (2005).

[13] Y. K. Gambhir, A. Bhagwat, and M. Gupta, *Phys. Rev.* **C 71**, 037301 (2005).

[14] P. R. Chowdhury, C. Samanta, and D. N. Basu, *Phys. Rev.* **C 73**, 014612 (2006).

[15] D. N. Poenaru and W. Greiner, In *Nuclear Decay Modes,* chapter 6, p. 275. Institute of Physics Publishing, Bristol, (1996).

[16] D. N. Poenaru, D. Schnabel, W. Greiner, D. Mazilu, and R. Gherghescu, *Atomic Data Nucl. Data Tables* **48**, 231 (1991).

[17] D. N. Poenaru and W. Greiner, *Physica Scripta* **44**, 427 (1991).

[18] D. N. Poenaru, M. Ivaşcu, and D. Mazilu, *J. Phys. Lett. (Paris)* **41**, L589 (1980);

[19] D. N. Poenaru, M. Ivaşcu, and D. Mazilu, *Comp. Phys. Communic.* **25**, 297 (1982).

[20] D. N. Poenaru, I. H. Plonski, R. A. Gherghescu and W. Greiner, *J. Phys. G* **32**, 1223 (2006).

[21] D. N. Poenaru, I. H. Plonski and W. Greiner, *Phys. Rev.* **C 74**, 014312 (2006).

[22] E. Roeckl, Ch. 5 in *Nuclear Decay Modes,* p. 237 Institute of Physics Publishing, Bristol (1996).

[23] A. Rytz, *Atomic Data Nucl. Data Tables* **47**, 205 (1991).

[24] G. Audi, A. H. Wapstra, and C. Thibault, *Nucl. Phys.* **A 729**, 337 (2003).

[25] G. Audi, O. Bersillon, J. Blachot, and A. H. Wapstra, *Nucl. Phys.* **A 729**, 3 (2003).

[26] V. E. Viola Jr. and G. T. Seaborg, *J. Inorg. Nucl. Chem.* **28**, 741 (1966).

[27] D. N. Poenaru and W. Greiner, *J. Phys. G: Nucl. Part. Phys.* **17**, S443 (1991).

FIGURE LEGEND

Figure 1 Experimental half-lives for α-decay of 162 even-even (E-E), 122 even-odd (E-O), 115 odd-even, and of 94 odd-odd (O-O) nuclides versus the neutron number of the daughter nucleus. The vertical bars correspond to spherical and deformed neutron magic numbers of the daughter nuclei $N_d = 50, 82, 126, 152, 162, 172$. They span a wide range of values between 10^{-7} and 10^{25} s. The particularly strong shell effect at $N_d = 126$ is very clearly seen.

Figure 2 Universal curves for α-decay of the same nuclides as in figure 1 in four groups of even-even, even-odd, odd-even and odd-odd nuclei.

Figure 3 The deviations of α-decay half-lives calculated with the SemFIS semiempirical formula from the experimental values for even-even (top) and odd-even (bottom) nuclei. The vertical bars correspond to spherical and deformed neutron magic numbers of the daughter nuclei $N_d = 50, 82, 126, 152, 162, 172$. There are nolonger large systematical discrepancies around magic numbers, because shell effects were taken into account by SemFIS. Calculations are performed with the new constants adjusted to fit the data of even-even nuclei.

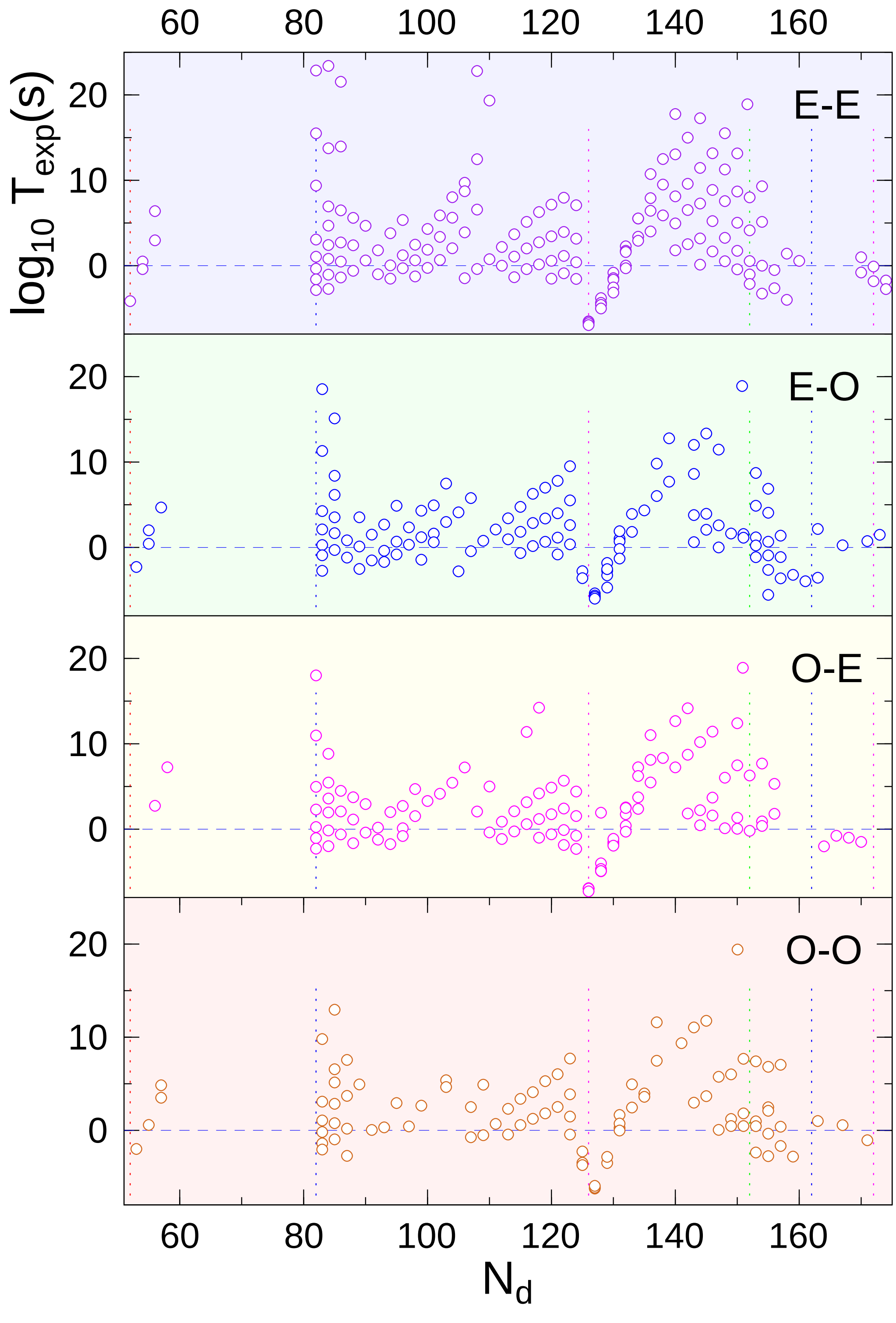

FIG. 1:

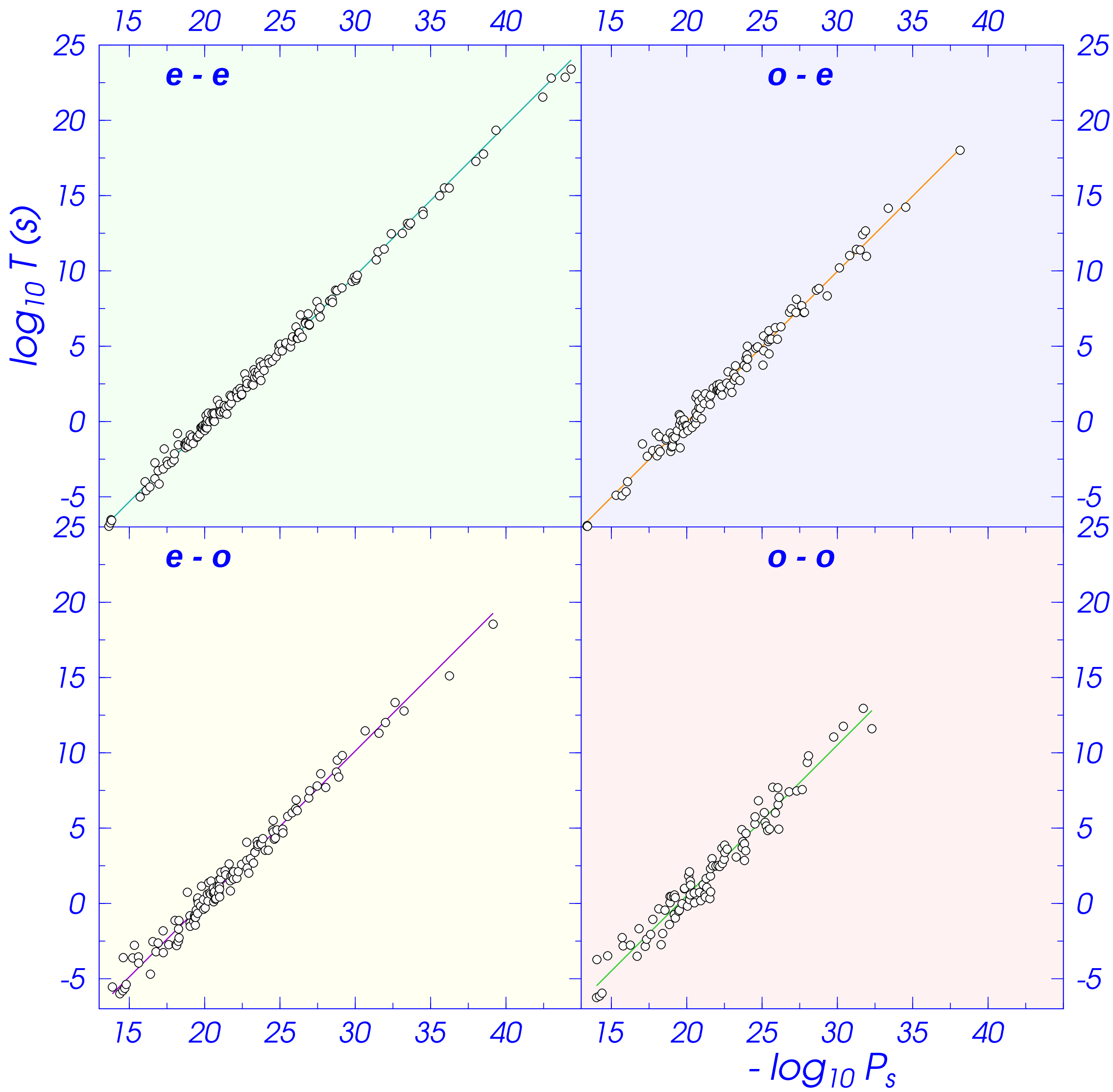

FIG. 2:

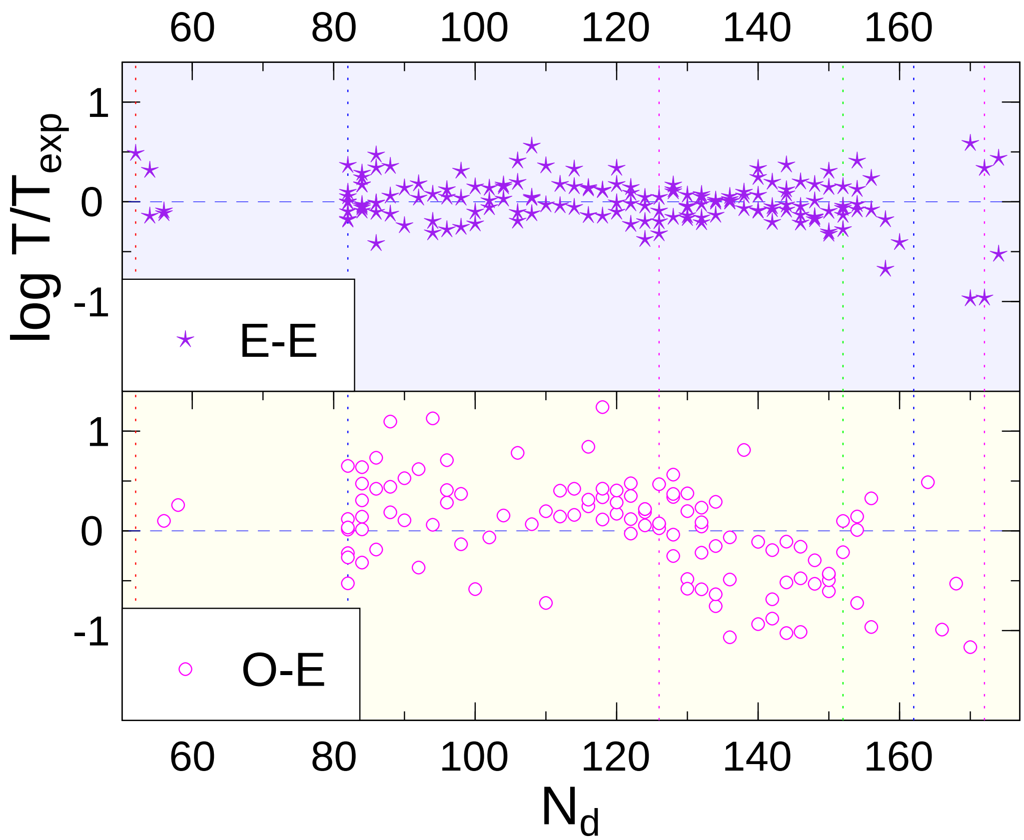

FIG. 3: